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//// ”MAGMA is a software package designed for computations in algebra, number theory,
//// algebraic geometry and algebraic combinatorics.”

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////Magma V2.16-10

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> R<l,r,XS,t,K2,k,n,j,s1,s2>:=PolynomialRing(Rationals(),10);

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>

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> ////Proof of Proposition 7:

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> a:=-2*k*l-48+12*l+12*k-8*XS+4*K2-4*t+s1;

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> b:=8+4*XS+t-K2+s2-2*k-2*l;

```

```

> H:=2*k^2-k*(4*XS+t-K2+8)+16*XS+2*t-2*K2;

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```

> a+(6-k)*b-H;

```

```

-k*s2 + s1 + 6*s2

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>

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>

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> ////Proof of Proposition 9:

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>

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> H:=2*k^2-k*(4*XS+t-K2+8)+16*XS+2*t-2*K2;

```

```

> G:=-2*k+4*XS+t-K2+8;

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```

> P1:=(2*l-G)*(k-r-2)-H;

```

```

> P2:=(2*l-G)*((r-4)*(k-r)+(r-2)*(k-r-2))-H*(2*r-6);

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> //a)

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```

> HH:=H-n*(k/2*(k/2-4)+(k/2-2)^2);

```

```

> GG:=G+n*(k-2);

```

```

> PP1:=(2*l-GG)*(k-r-2)-HH;

```

```

> E:=Evaluate(2*PP1,[k+2,k/2,XS,t,K2,k,n,j,s1,s2]);

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```

> Coefficients(E,k);

```

```

[

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```

-16*XS + 16,

```

```

4*XS + t - K2 - 2*n - 4

```

```

]

```

```

>

```

```

> //b)

```

```

> Eb:=Evaluate(2*P2,[k+4,k/2+2,XS,t,K2,k,n,j,s1,s2]);

```

```

> Coefficients(Eb,k);

```

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[

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```

32*XS,

```

```

-16*XS,

```

```

4*XS + t - K2 - 8

```

```

]

```

```

>

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```

> //c)

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```

> Ec1:=Evaluate(2*P2,[k-2,k/2,XS,t,K2,k,n,j,s1,s2]);

```

```

> Coefficients(Ec1,k);

```

```

[

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160*XS + 16*t - 16*K2 - 96,

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-48*XS - 8*t + 8*K2 + 32,

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4*XS + t - K2 - 4

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]

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>
> HH:=H-(k/2-2)^2;
> GG:=G+k/2-2;
> PP1:=(2*I-GG)*(k-r-2)-HH;
> Ec2:=Evaluate(2*PP1,[k-2,k/2-2,XS,t,K2,k,n,j,s1,s2]);
> Coefficients(Ec2,k);
[
  -32*XS - 4*t + 4*K2 + 8,
  4*XS + t - K2 + 2
]
>
> PP2:=(2*I-GG)*((r-4)*(k-r)+(r-2)*(k-r-2))-HH*(2*r-6);
> Ec3:=Evaluate(2*PP2,[k-2,k/2,XS,t,K2,k,n,j,s1,s2]);
> Coefficients(Ec3,k);
[
  160*XS + 16*t - 16*K2 - 128,
  -48*XS - 8*t + 8*K2 + 44,
  4*XS + t - K2 - 5
]
>
> //d)
> HH:=H-n*(k/2-2)^2;
> GG:=G+n*(k/2-2);
> PP1:=(2*I-GG)*(k-r-2)-HH;
> E:=Evaluate(2*PP1,[k+j,k/2-2,XS,t,K2,k,n,j,s1,s2]);
> Coefficients(E,k);
[
  -32*XS - 4*t + 4*K2 + 8*n,
  4*XS + t - K2 - 2*n + 2*j + 8
]
>
> //e)
> E:=Evaluate(P1,[k/2+r-2,r,XS,t,K2,k,n,j,s1,s2]);
> C:=Coefficients(E,r);
> C;
[
  -8*XS + k^2 - 10*k + 24,
  4*XS + t - K2 - k + 8,
  -2
]
> b:=C[2];
> c:=C[1];
> f:=b^2+8*c-(4*(k/2-2)-b)^2;
> 1/4*f;
4*XS*k - 32*XS + t*k - 4*t - K2*k + 4*K2
> Coefficients(1/4*f,k);
[
  -32*XS - 4*t + 4*K2,
  4*XS + t - K2
]
>
> //f)

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```

> HH:=H-n*2*(k/2-1)*(k/2-3);
> GG:=G+n*(k-4);
> PP1:=(2*l-GG)*(k-r-2)-HH;
> Ef1:=Evaluate(2*PP1,[k,k/2-1,XS,t,K2,k,1,j,s1,s2]);
> Coefficients(Ef1,k);
[
  -24*XS - 2*t + 2*K2 + 20,
  4*XS + t - K2 - 2
]
>
> Ef2:=Evaluate(2*P2,[k,k/2+1,XS,t,K2,k,n,j,s1,s2]);
> Coefficients(Ef2,k);
[
  80*XS + 4*t - 4*K2 - 96,
  -32*XS - 4*t + 4*K2 + 48,
  4*XS + t - K2 - 8
]
>
> //g)
> E:=Evaluate(P1,[k/2+r-2,r,XS,t,K2,k,n,j,s1,s2]);
> C:=Coefficients(E,r);
> C;
[
  -8*XS + k^2 - 10*k + 24,
  4*XS + t - K2 - k + 8,
  -2
]
> b:=C[2];
> c:=C[1];
> f:=b^2+8*c-(4*(k/2-1)-b)^2;
> 1/4*f;
4*XS*k - 24*XS + t*k - 2*t - K2*k + 2*K2 - 6*k + 28
> Coefficients(1/4*f,k);
[
  -24*XS - 2*t + 2*K2 + 28,
  4*XS + t - K2 - 6
]
>
>
>
>
> ////Proof of Theorem 1
>
> //b)
> z:=16*XS/(4*XS+t-K2-8);
> a:=Coefficients(Eb,k)[3];
> b:=Coefficients(Eb,k)[2];
> z+b/(2*a);
2*$.3/($.3 + 1/4*$.4 - 1/4*$.5 - 2)
> Evaluate(Eb,k,z);
32*$.3
>
>

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```

> //c)
> z:=4+16*XS/(4*XS+t-K2-4);
> a:=Coefficients(Ec1,k)[3];
> b:=Coefficients(Ec1,k)[2];
> z+b/(2*a);
2*$.3/($.3 + 1/4*$.4 - 1/4*$.5 - 1)
> Evaluate(Ec1,k,z);
32*$.3 - 32
>
> //c')
> z:=4+(16*XS-4)/(4*XS+t-K2-5);
> a:=Coefficients(Ec3,k)[3];
> b:=Coefficients(Ec3,k)[2];
> z+b/(2*a);
(2*$.3 - 1/2)/($.3 + 1/4*$.4 - 1/4*$.5 - 5/4)
> Evaluate(Ec3,k,z);
32*$.3 - 32
>
> //f)
> z:=2+(16*XS-16)/(4*XS+t-K2-8);
> a:=Coefficients(Ef2,k)[3];
> b:=Coefficients(Ef2,k)[2];
> z+b/(2*a);
(2*$.3 - 2)/($.3 + 1/4*$.4 - 1/4*$.5 - 2)
> Evaluate(Ef2,k,z);
32*$.3 - 32
>
>
>
> ///Proof of Proposition 2
>
> alpha:=-s1+2*k^2-k*(4*XS+t-K2+8)+16*XS+2*t-2*K2;
> beta:=-2*l-2*k+4*XS+t-K2+8+s2;
> gamma:=(alpha+(k-10)*beta)/8;
> gamma - (15+K2-t-3*XS-1/4*(k-10)*(l-10));
1/8*k*s2 - 1/8*s1 - 5/4*s2
> beta + gamma + (-XS+1+1/4*(k-2)*(l-2));
1/8*k*s2 - 1/8*s1 - 1/4*s2

```

///Algorithm for the proof of Theorem 3:

for M in [-18..-7] do for k in [12..28 by 2] do

s:=[0];

for t in [0..11] do for N4 in [0..11] do

l:=10+4/(k-10)*(15+M-t-N4);x:=1+1/4*(k-2)*(l-2)-N4;

c0:= 1 in Integers();

c1:= 1 ge k/2;

c2_1:= 1 ne Integers()!(k/2) or (t eq 2 and N4 eq 0);

```

c2_2:=(t ne 2 or N4 ne 0) or l eq Integers()!(k/2);
c3:=l ne Integers()!(k/2+2) or (t ge N4 and (t eq N4 or N4 gt 1));
c4:=(l ne k-2 or t ne 0) or (k/2) in {!x in Integers()|x/2 in Integers()!};
c5:=l ge k-2 or (l-k/2) in {!x in Integers()|x/2 in Integers()!};
c6:=(t ne 1 or N4 ne 0) or l eq k-2;

if c0 and c1 and c2_1 and c2_2 and c3 and c4 and c5 and c6 and (x gt s[1]) then s:=[x,M,k,l,t,N4];
end if;

end for;end for;

print s;

end for;end for;

//Output:

[ 16, -16, 12, 8, 0, 0 ]
[ 22, -16, 14, 9, 0, 0 ]
[ 22, -16, 16, 8, 2, 0 ]

...

[ 21, -15, 12, 10, 0, 0 ]
[ 0 ]
[ 29, -15, 16, 10, 0, 0 ]
[ 29, -15, 18, 9, 2, 0 ]

...

[ 26, -14, 12, 12, 0, 0 ]
[ 28, -14, 14, 11, 0, 0 ]

...

[ 31, -13, 12, 14, 0, 0 ]
[ 27, -13, 14, 11, 0, 1 ]
[ 28, -13, 16, 10, 1, 1 ]
[ 37, -13, 18, 11, 0, 0 ]
[ 37, -13, 20, 10, 2, 0 ]

...

[ 36, -12, 12, 16, 0, 0 ]
[ 34, -12, 14, 13, 0, 0 ]
[ 36, -12, 16, 12, 0, 0 ]

...

[ 41, -11, 12, 18, 0, 0 ]
[ 37, -11, 14, 14, 0, 0 ]
[ 35, -11, 16, 12, 0, 1 ]

```

[36, -11, 18, 11, 1, 1]

...

[46, -10, 12, 20, 0, 0]

[40, -10, 14, 15, 0, 0]

[35, -10, 16, 12, 1, 1]

[0]

[46, -10, 20, 12, 0, 0]

[46, -10, 22, 11, 2, 0]

...

[51, -9, 12, 22, 0, 0]

[43, -9, 14, 16, 0, 0]

[43, -9, 16, 14, 0, 0]

[45, -9, 18, 13, 0, 0]

...

[56, -8, 12, 24, 0, 0]

[46, -8, 14, 17, 0, 0]

[43, -8, 16, 14, 1, 0]

[44, -8, 18, 13, 0, 1]

[45, -8, 20, 12, 1, 1]

...

[61, -7, 12, 26, 0, 0]

[49, -7, 14, 18, 0, 0]

[42, -7, 16, 14, 1, 1]

[44, -7, 18, 13, 1, 1]